Measurement of Directional Wave Spectrum with a Modular Acoustic Velocity Sensor

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***Abstract*- Directional wave spectra, valuable to describe the sea-state, can be computed as the correlation of horizontal velocity with pressure measured at a single point. Pressure was used instead of the vertical component of velocity for the correlation since the deployment height of 70 cm above the bottom attenuated the vertical wave velocity component. A Modular Acoustic Velocity Sensor (MAVS) with integral pressure sensor was deployed on the bottom in 10 meters depth in Buzzards Bay, Massachusetts during February 2000 in conjunction with the ASIS Multi-Spar Buoy, providing surface wave measurements, and other wave sensing instruments. MAVS was attached to a bracket on the corner of a sled resting on the bottom that contained a fan beam Acoustic Doppler Current Profiler (ADCP) for measurement of directional wave spectra from beneath the surface. The purpose of the experiment was to observe fetch-limited waves. The deployment site, near Nye Ledge, is about 7 km from shore in all directions except southwest. Waves with periods ranging from 5 to 12 seconds were measured propagating north to northeast when strong winds blew from the southwest. Velocities and pressure were sampled at 1.4 Hz for 14 minutes every 4 hours. The observations of velocity were corrected for tilt and compass orientation of the sensor and stored as Cartesian components of velocity along with digitized output of a 20 decibar Omega pressure sensor. Although the deployment of the instrument was 25 days, the data record from MAVS was limited to 15 days by battery and data capacity. This technique of directional wave spectral measurement from a simple vector current meter with pressure sensor demonstrates a capability that can be used in moored arrays for wave refraction studies over topography or for shoaling waves and fetch limited waves as a function of distance from the shore. The low cost of the instrument used in this experiment permits such an array to be deployed with moderate cost.**

I. INTRODUCTION

A pressure sensor combined with a two horizontal component velocity sensor can in principal permit correlation of pressure with horizontal velocity components to produce directional wave spectra. If the current meter is fixed near the bottom, only the horizontal velocities are used and the pressure is the correlating scalar. If the current meter is fixed near the surface, either pressure or vertical velocity can be used as the correlating scalar. Since it is relatively easy to fix a current meter near the bottom, a test was made of how well it worked in practice with a MAVS2 acoustic current meter [1] and Omega 30 psi pressure sensor [2] fixed 70 cm above bottom in 10 meters of water.

A. *Wave Velocities Near the Bottom*

Positioning the current meter near the bottom filters the wave spectra two ways. In deep water, high wave number waves are attenuated more rapidly with depth than low wave number waves.

$$u = a\sigma e^{\kappa(z-h)}\cos(\kappa x - \sigma t) \qquad (1)$$

$$w = a\sigma e^{\kappa(z-h)}\sin(\kappa x - \sigma t) \qquad (2)$$

where u is the horizontal velocity, w is the vertical velocity, a is the wave amplitude, σ is the frequency of the wave, κ is the wave number, h is the depth of the water, and z is the height above bottom. In shallow water, the velocity does not attenuate with depth but the low wave number waves are simply not present (except where they shoal from a deeper source region).

$$u = \frac{a\sigma}{\kappa h}\cos(\kappa x - \sigma t) \qquad (3)$$

$$w = a\sigma\left(1 + \frac{(z-h)}{h}\right)\sin(\kappa x - \sigma t) \qquad (4)$$

The vertical velocity, equal in magnitude but out of phase with the horizontal velocity in the deep water case, is linearly attenuated with height off the bottom in the shallow water case [3].

Very near the seabed, the wave velocity is affected by friction with the bottom. This wave boundary layer, WBL, establishes itself during the half period when the wave induced current is flowing in one direction and then a new WBL is established during the other half period when the current is reversed. However, this region is very thin, even for long period waves.

$$\delta_w = \sqrt{\frac{2\upsilon}{\sigma}} \qquad (5)$$

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where δ_w is the WBL thickness, ν is the kinematic viscosity, and ω is the radian wave frequency. The WBL is typically thinner than 0.5 cm. This means that velocity measurements near the bottom are unlikely to be in the wave boundary layer. Thus, the flow measurements attributable to the waves are unaffected by the frictional nature (no slip condition) of the bottom [4].

B. *Wave Pressure Near the Bottom*

Beneath the wave crest, the pressure is higher than beneath the trough. In deep water, the pressure variation due to waves is

$$\Delta p = \rho g \frac{H}{2} e^{\kappa(z-h)} \cos(\kappa x - \sigma t) \qquad (6)$$

where Δp is the departure from hydrostatic pressure due to the wave, ρ is the density, g is gravity, H is the crest to trough wave height, and the remaining symbols are as in Eq. (1&2). Note that the pressure signal attenuates with depth beneath the surface as does the horizontal velocity. In shallow water

$$\Delta p = \rho g \frac{H}{2} \cos(\kappa x - \sigma t) \qquad (7)$$

and the pressure signal remains unattenuated to the bottom [3].

II. EXPERIMENT

A deployment of MAVS2 in Buzzards Bay, MA from February 7 to March 8, 2000 was part of a shallow-wave experiment at a site 10 km from shore in every direction but southwest. It was cold, -8°C on the day of launch and close to freezing during the next week. In fact, half the battery and memory capacity of MAVS was wasted while new fuel was obtained to replace diesel oil that had gelled in the ship's fuel tanks. Stratification was not dynamically significant. Other wave sensors included a fan beam Acoustic Doppler Current Profiler, a wave gauge supporting multispar buoy, and a Wave Rider accelerometer buoy. Data from these sources is not yet available.

A. *Program*

1) *Burst Sample Mode*: MAVS was programmed to measure and record at 700 ms intervals for 1200 samples every 240 minutes, yielding 14 minutes of measurements every 4 hours. Power, provided by a 22-volt lithium battery, was 24 ma during the sampling burst and 14 ma between bursts. The capacity of the lithium battery is 5 ah so that the experiment had a projected deployment time of 342 hours. That was nearly completely dominated by the power drain between bursts. (It was partly this experience that led to the development of a low power option for MAVS2 [5] that would have increased the deployment time to 140 days, a factor of ten.)

2) *Data Storage*: Each measurement produces an 18-byte record. The format is: time from day of month to seconds, temperature, pressure, tilt, and the three Cartesian coordinates of velocity. Velocity is rotated into earth coordinates, first by rotation about the roll axis to remove the effects of pitch, second by rotation around the pitch axis to remove the effects of roll, and finally by rotation around the vertical axis to align the Cartesian axes with east, north, and up. Each burst produces 21,200 bytes of data. The duration of the deployment was limited by battery life to 14 days. With 6 bursts recorded a day, the 14 days of the planned deployment would have produced 1.8 Megabytes of data. MAVS2 has storage capacity for 2.1 Megabytes of data so this was a good match.

A. *Instrumentation*

1) *Velocity Sensor*: The Modular Acoustic Velocity Sensor has two rings that support eight piezoceramic transducers. Each pair of transducers, one on each ring, defines an acoustic path along which a component of flow is measured. Since no path is horizontal, all being inclined 45° to the plane of the rings, the wake of the rings in horizontal flow does not cross an acoustic path. Out of plane flow is also acceptable because the rings are faired in the direction of the acoustic axes to minimize flow obstruction for all directions. Resolution of the velocity measurement is 0.5 mm/s with directional precision limited by the compass to 2°. There is no hysteresis or frictional sticking in the velocity measurement. Drift of the zero point of the velocity is typically 0.5 cm/s, inconsequential to wave measurements.

Pressure is measured with a strain gauge sensor with an analog, 5-volt, output. A 12-bit A/D converter on the MAVS2 Tattletale 4A processor [6] digitizes the analog output. With a full-scale range in the Omega 30-psi sensor of 20 meters in hydrostatic head, a single bit from the digitizer is 5mm of head. Electronic noise was about a single bit so that short-term pressure variations due to waves could be measured to 0.01 decibar.

2) *Mounting*: MAVS is a cylinder 83mm in diameter and 433mm long with a sensor extending downward 414mm from the lower end. This cylinder was mounted on an aluminum channel extended horizontally outboard from the instrumentation sled that sat on the seafloor. The MAVS sensor volume was 70 cm above the bed of the sled and presumably above the bottom. The pressure sensor port was 100 cm above the bed of the sled. Flow was slightly obstructed in some directions by other instruments on the sled. The MAVS sensors were raised as high above the bottom as structure on the sled permitted.

B. *Observations*

Although the main battery was dead upon recovery, the datafile was still readable and had 1.1 Megabytes of data. Streaming these to a PC took about 6 hours. Quick inspection showed the temperature to be about 1°C and pressure to be about 11.5 decibars. Velocities were occasionally as strong as 15 cm/s but were typically less than 5 cm/s.

1) *Two Wave Events:* Evidence of waves in the form of pressure variations was observed on 2/9/00 and 2/14/00. Fig. 1 shows these events. Each event lasted only one day but the variance of pressure and the variance of velocity indicate waves. Wind on these two days was southwest at 15 to 25 knots.

2) *Time Series:* Fig. 2 shows a 5 minute section of one of the 14 minute bursts on 2/14/00 and exhibits the correlation between pressure and east component of velocity that is characteristic of a wave signal. Since the pressure and velocity components are in phase, they correspond to a wave moving east. (Constant phase in Eq. 3&7 is x increasing as t increases.) There are 37 cycles of pressure in 300 seconds of this record corresponding to a period of 8.1 seconds. The records are nearly monochromatic with groups of more than 6 cycles.

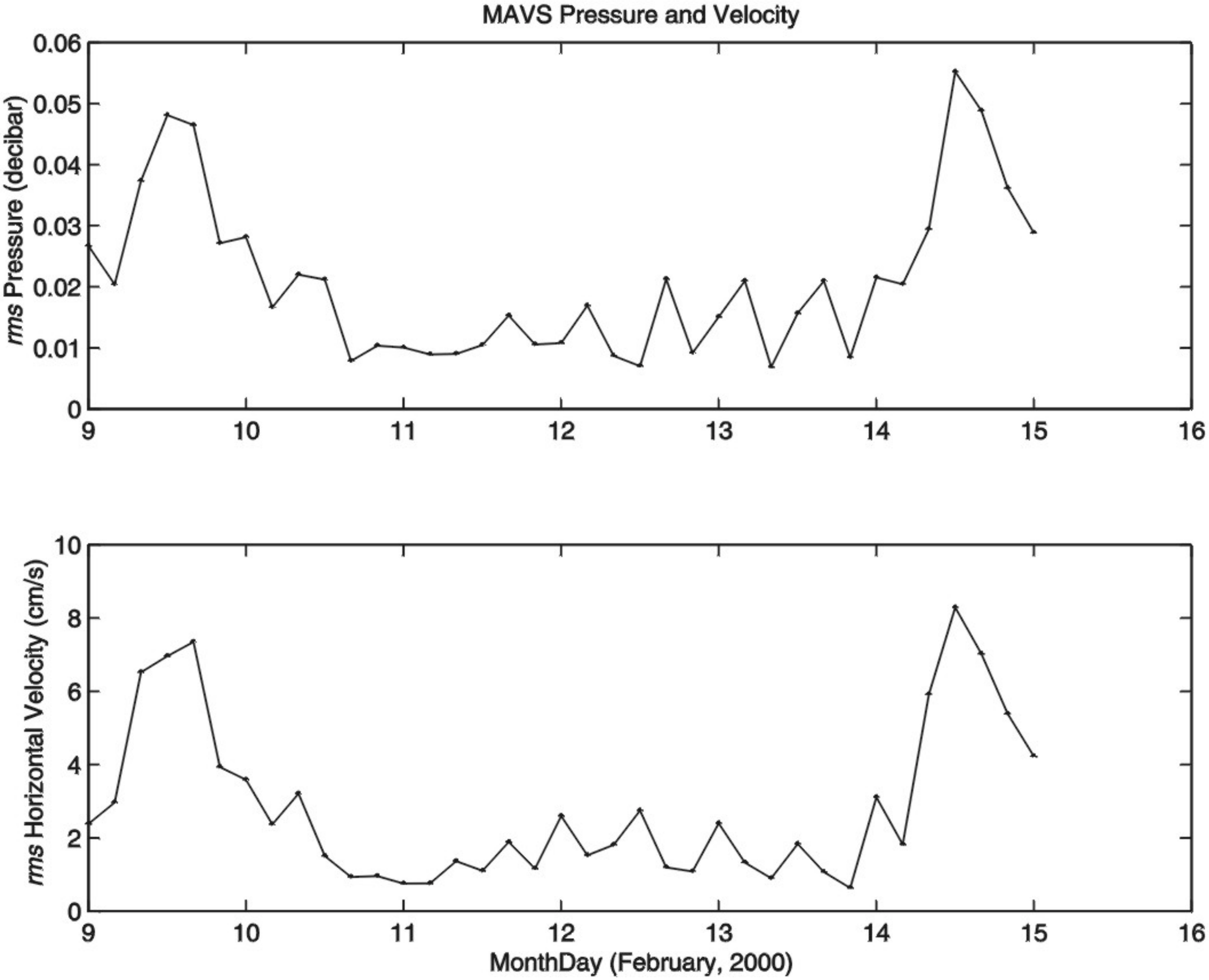


Fig. 1. Pressure variance and velocity variance from MAVS.

The pressure record is offset by its mean but the velocity is the actual velocity corresponding to a westerly flow of 11 cm/s. Included in each record, but particularly the velocity record, are fluctuations resulting from the boundary layer flow. The velocity fluctuations result from turbulent momentum exchange between the flow above the bottom and the slower flow nearer the bottom [7]. There are undoubtedly also eddies with vertical vorticity caused by bottom obstructions including the presence of the sled upon which the MAVS was mounted.

3) *Spectrum:* The spectrum of the burst, a portion of which is displayed in Fig. 2, is displayed as Fig. 3. This shows the spectrum of pressure and the spectrum of horizontal velocity as a function of frequency. It also shows the coherence of the pressure with each of the two horizontal axes of velocity, east and north.

III. ANALYSIS

The spectrum of a signal is derived from a finite time series of observations. The burst is such a piece. The spectral resolution varies inversely with the length of the burst. However, resolution can be traded for accuracy by breaking the piece into shorter segments [8]. Each of these produces a spectrum with restricted resolution. These spectra can be averaged to produce a spectrum with smaller error bars. The degrees of freedom are twice the number of independent pieces that are averaged.

A. Windowing: A piece of data with constant weighting when transformed exhibits higher frequency components than may have been present in a longer piece of data due to the abrupt interruption of the ends. To reduce this effect, a window with tapered ends can multiply the data piece. Cosine, Hanning, Hamming, and other windows can be used. Of course, the windowing process loses data at the ends of each piece. If overlapping pieces are windowed, this loss can be minimized.

The window that was used in this analysis was the Blackman-Harris window where the weighting w for the mth point in the interval of n points is:

$$w = 0.36 - 0.49\cos(2\pi m/n) + 0.14\cos(4\pi m/n) - 0.01\cos(6\pi m/n) \quad (8)$$

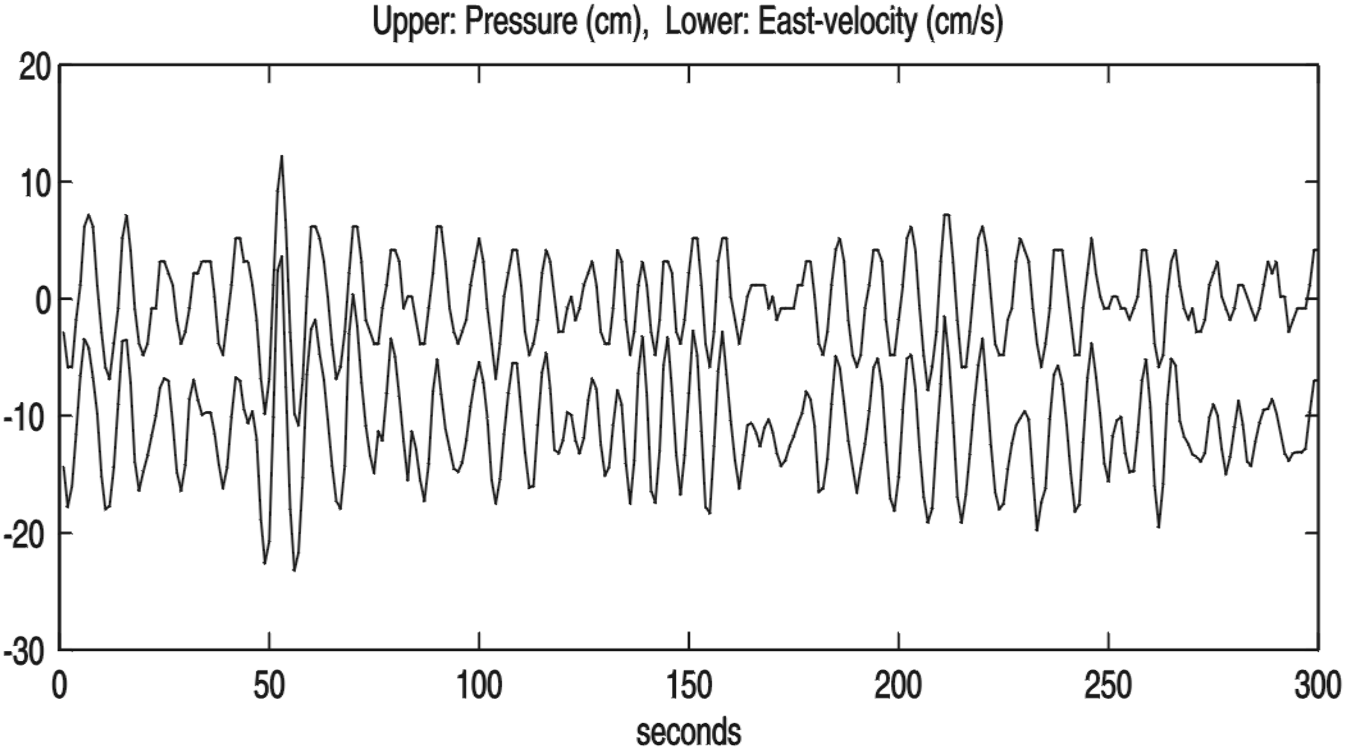
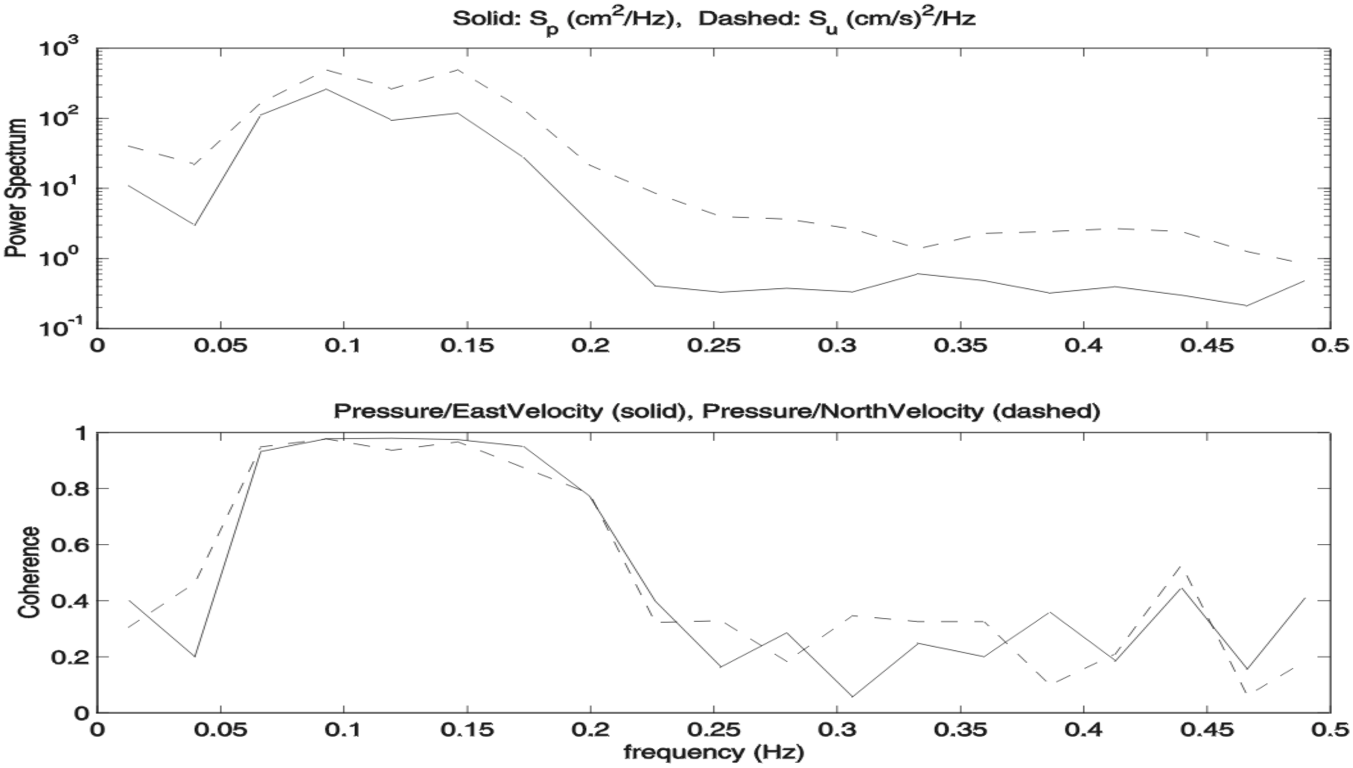


Fig. 2. Time series of East-velocity and pressure.


Solid: S_p (cm^2/Hz), Dashed: S_u $(cm/s)^2$/Hz

Pressure/EastVelocity (solid), Pressure/NorthVelocity (dashed)

Fig. 3. Spectrum of pressure in the upper solid curve and spectrum of horizontal velocity in the upper dashed curve. Coherence of pressure with the east component of velocity in the lower solid curve and coherence of pressure with north component of velocity in the lower dashed curve.

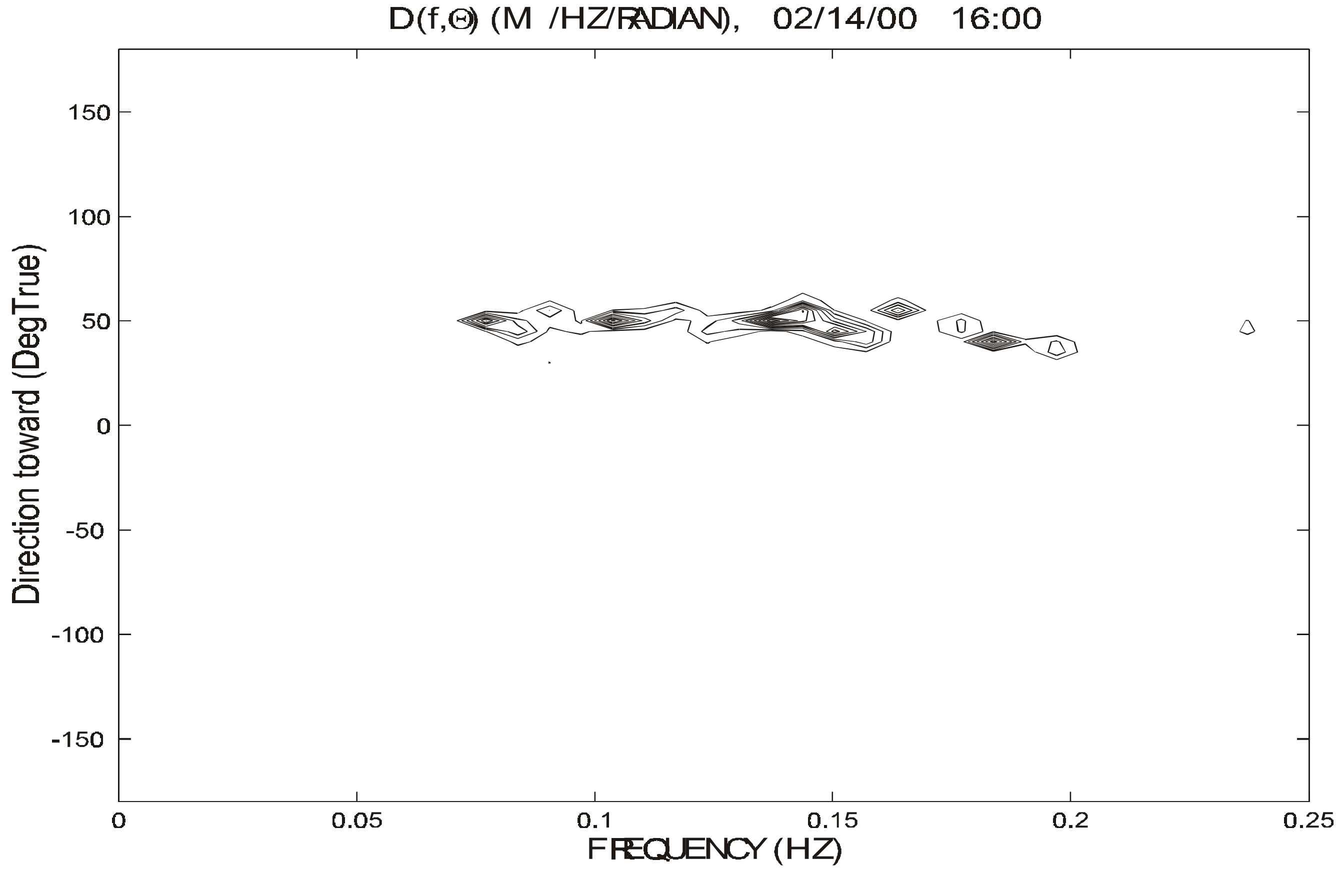


Fig. 4. Directional wave spectrum for the case shown in Fig. 1, 2, and 3.

B. *Spectrum:* The spectrum of the n point time series is determined with a Fast Fourier Transform, FFT, of the windowed and detrended data points. Detrending simply removes a linear fit to the points since the spectrum is dominated by a linear component to the data. The FFT produces a spectrum with the same number of values as data points in the windowed time series.

C. *Band Averaging:* Averaging across a spectral band by summing the squares of the spectral amplitudes and dividing by the number of spectral points used, averages the power in the spectrum for a less noisy but lower resolution spectrum. In the data displayed here, the spectrum was averaged over 32 bands. Since the original data piece had 1200 points and thus there were 1200 spectral coefficients, the band averaging with 32 bands improved the signal to noise ratio in each band by about a factor of 6, the square root of 34, the number of spectral points summed in each band.

D. *Cross Spectrum:* The cross spectrum of pressure and velocity is the wave spectrum. The power spectrum for velocity and the power spectrum for pressure are formed from the FFT of the windowed time series and band averaged as before but a cross spectrum is formed by multiplying the complex conjugate of the pressure spectrum by the velocity spectrum and band averaging the product. The coherence of the two signals is the absolute value of the cross spectrum normalized by the square root of the product of the individual spectra.

E. *Directional Spectrum:* The vector components of velocity individually form cross spectra with pressure and each is one component of the directional wave spectrum. In the case represented in Fig. 1, 2, and 3, a contour plot, Fig. 4., shows the amplitude of the cross spectrum as a function of frequency and direction. The peak spectral feature for this case is at about 50° magnetic and runs from 0.07 Hz to 0.2 Hz..

F. Significant Wave Height: Fig. 5 shows the spectral energy of the wave as a function of frequency. The significant wave height for this distribution of waves is 42 cm. It is greatest at 0.14 Hz or a 7 second period. The swell at lower frequency has less amplitude.

G. Interpretation of Results: The band of waves from 0.06 Hz to 0.2 Hz where the coherence is above 0.6 is the wave band of the velocity and pressure cross spectrum. This is the wave spectrum. In this band, the waves are northeast in propagation direction. A southerly wind generates a northerly wave and as the fetch increases, the amplitude increases and the frequency decreases. In the upper part of Buzzards Bay, the fetch is limited to 10 km in all directions but southwest. Even with a steady strong wind from the south, there are variations in wind direction. The more westerly wind component has a longer fetch and can generate larger waves. These propagate in the only direction where the fetch is greater than 10 km, the southwest direction and thus are a northeast wave. This is the 0.10 Hz or 10 second period wave. In 11 meters depth, this is a shallow water wave.

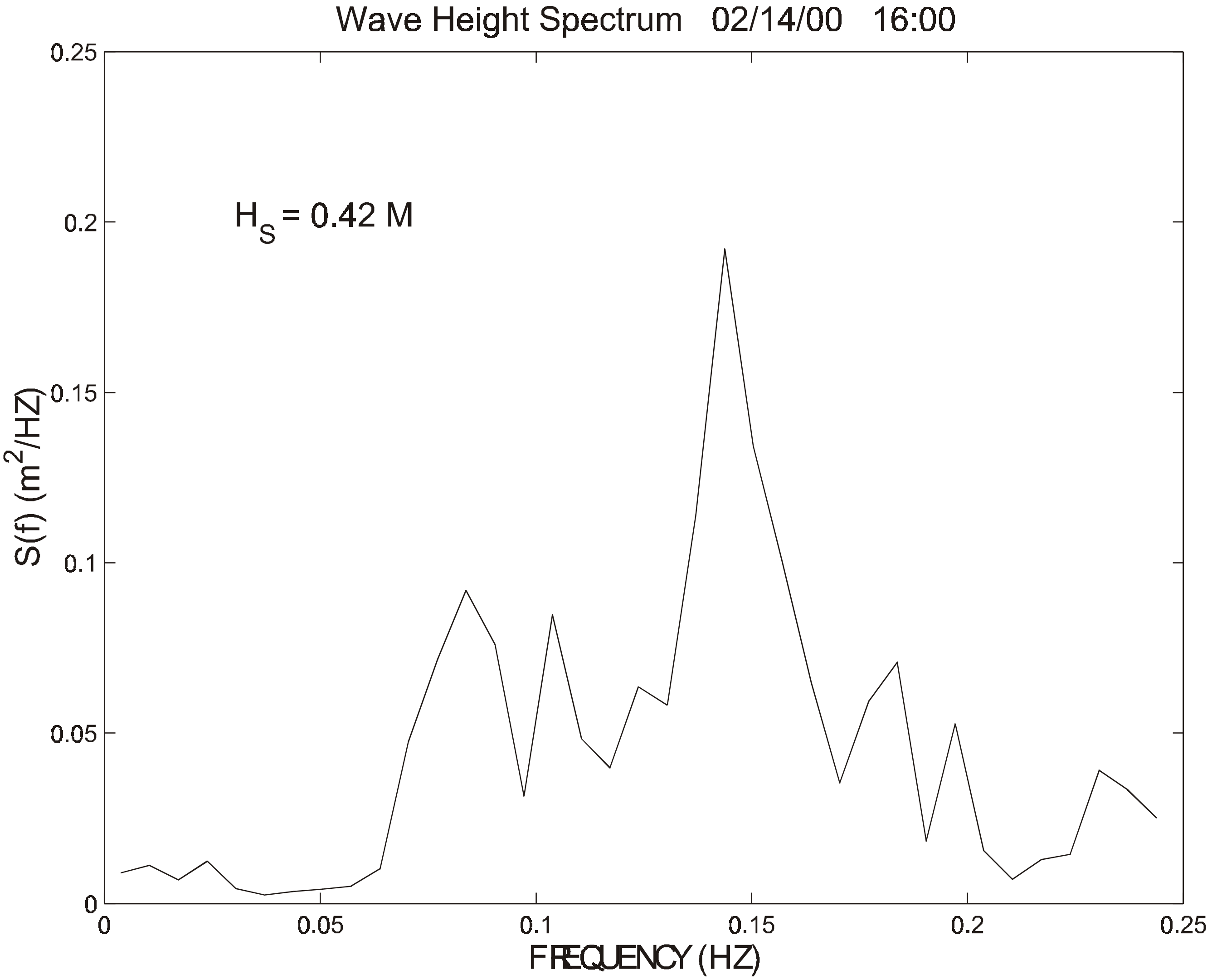


Fig. 5. Wave height spectrum for the case shown in Fig. 1-4. The peak at 0.14 Hz is the 7 second period wave with the 0.42 m significant wave height.

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