The Single Axis Sample Volume of the BASS Rake Acoustic Current Sensor

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Abstract - **The BASS Rake is a differential travel time acoustic current meter designed to make spatially and temporally dense velocity profile measurements in the continental shelf wave bottom boundary layer (WBBL). The high levels of bottom shear stress common to the wave boundary layer are important contributors to the sediment entrainment process and strongly enhance the turbulent dissipation of energy in overlying steady boundary layers. The geometry of the acoustic axes of earlier BASS sensors made accurate determination of the sample volume over which velocity is averaged during a measurement unnecessary. However, the arrangement of BASS Rake acoustic axes, the small vertical scale of velocity profile curvature in the wave boundary layer, and the proximity of the fluid-sediment interface, make an accurate understanding of the averaging volume and the error it can introduce into a velocity measurement essential to the determination of sensor accuracy.**

I. INTRODUCTION

Historically, it has not been necessary to determine the actual volume of water sampled by a single BASS acoustic axis. The standard arrangement of the four crossing acoustic axes necessarily averages the velocity field over a volume that is large compared to any reasonable estimate of the volume sampled by each path. Additionally, BASS sensors are always placed 30 *cm* or more above the bottom. This is done so that the most energetic turbulent scales present in the volume are not hidden by the implicit spatial average. In consequence, the sample volume of an individual path never interacts with the boundary [4]. The development of the BASS Rake has made it necessary to clearly define the single axis sample volume and to understand the effect it has on a measurement of the wave and steady components of the flow.

The necessity arises both because of the fine vertical resolution of horizontal velocity the new geometry of the BASS Rake is designed to provide and because of the intended proximity of the bed during operation. In BASS, differential arrival time is measured from a particular zero crossing of the received wave form. The wave form is the sum of signals arriving along multiple rays with disparate lengths and along-ray fluid velocities, both of which affect arrival phase. It can be shown, by summing these sine wave bursts, that the measured velocity is an average, weighted by signal strength, of the integrated fluid velocities along all of the constructively contributing rays. A pair of transducers placed at a particular height above bottom will report a fluid velocity averaged over the cross-axis extent of the ray bundle. The averaging smooths the velocity profile, limiting resolution. More seriously, the averaging produces systematic errors when the profile is non-linear. For example, the average over some range of a logarithmic profile will always be low because the mean slope of the profile is greater above the transducer centerline than below it. The error increases as the boundary is approached. An additional error occurs if the sample volume and the boundary overlap. That error will depend on the geometry of the sample volume, its proximity to the bottom, and the nature of the signal's interaction with the bed. Quantifying these effects is essential to a full characterization of the sensor response.

A formal mathematical and physical description of the sample volume of a single acoustic axis is presented in Section II. In Section III a simplified, parametric model is described and shown to yield substantially the same results. The differences are well below the measurement accuracy of the sensor and the computational overhead is reduced by up to four orders of magnitude. The parametric model is used to estimate the measurement error for a canonical continental shelf WBBL in Section IV.

II. FORMAL DESCRIPTION

The brief description of acoustic propagation in a scattering medium presented here derives from the published work of many authors who have contributed to our understanding of wave propagation. A presentation of this rich historical background with the details of the current derivation is not possible here. However, a broad survey of the classical and modern literature, a full, formal mathematical derivation and physical interpretation, and an extended bibliography, can be found in [3]. The response of the instrument to the turbulent component of the flow will not be considered in this paper, but is also explored in [3].

The concentric rings of the diffraction pattern produced by a flat wave front incident on a round aperture in a large, flat sheet will be familiar to most readers. The process can be understood in terms of Huygens' Principle, which holds that each point on a wave front acts as a spherical radiator. The field at any subsequent point is the sum of the waves arriving from each radiator. Because the radiators are at different distances the waves will arrive with different phases and add either construc-

This work was supported by AASERT funding under ONR grant N00014-96-1-0953 while the author was a doctoral student at the Woods Hole Oceanographic Institution. WHOI Contribution No. 9733.

tively or destructively to produce the diffraction rings. The field pattern is unchanged when the aperture is replaced with a flat, round radiator, the BASS Rake transducer. The face of the receiving transducer defines the portion of the transmitted field that is of interest and the reciprocity of transmission and reception produces symmetry across a midpoint plane between the two transducers. It follows from consideration of the diffraction patterns at sequential planes perpendicular to the centerline axis that the regions of the transmit/receive field contributing either constructively or destructively to the received signal are concentric, ellipsoidal shells. The BASS measurement is a weighted average of the fluid velocity field over these shells [2, 3].

There are several problems with Huygens' physical description as originally formulated, but these were resolved by Fresnel, based on an understanding of wave interference. Fresnel derived a mathematical description of the modified Huygens' Principle now known as the Fresnel-Kirchoff Formula. In this century the formulation has been further modified to describe wave propagation in a scattering medium [2]. For the case of a BASS Rake acoustic velocity axis, that description is given by

$$\psi'(L,y,z) = ik\int_0^L dx_1 \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} n'(x_1,y_1,z_1) \cdot \left[\frac{\exp\left(-\rho^2/2\sigma^2\right)}{2\pi\sigma^2}\right] dy_1 dz_1 \quad (1)$$

where $\sigma^2 = \frac{i(L-x_1)}{k}$. ψ' denotes fluctuations in the eikonal, ψ_o, of a plane wave, $E_o = A_o e^{iS_o} = e^{\psi_o}$, propagating through a volume with fluctuations, n', in the index of refraction. For the case of the BASS Rake the field n' is equivalent to the velocity field normalized by the (constant) speed of acoustic propagation [3]. Fluctuations in the eikonal on the plane $x = L$, which includes the receiver, are of interest. The transmitter is located at $(0, y, z)$. ρ is radial distance from the centerline and k is the acoustic wave number. While several simplifications were used in the derivation, they are consistent with the characteristics of the boundary layer and the behavior of BASS transducers [2, 3].

Physically, the real component of the eikonal is simply the logarithm of the ratio of A, the actual wave amplitude at the receiver, to A_o, the amplitude of the wave at the receiver had it not been altered by scattering. The imaginary component is the phase difference between those two signals. In the absence of severe attenuation, the velocity measurement depends only on the imaginary portion of the eikonal. Early or late arrival of the signal due to the velocity of the fluid is the basis of the BASS measurement and this is precisely the phase change described by the eikonal. Equation 1 clearly shows the dependence of that measurement on the whole field of velocity variations, $n'(x_1, y_1, z_1)$, not just along the centerline between the transducers [2, 3].

The term in brackets describes the defractive smearing of that velocity field, modulating the acoustic signal and producing a velocity measurement that is a weighted average over the sample volume. Note that the exponential oscillates rapidly as distance from the centerline increases. This is the mathematical manifestation of the ellipsoidal shells or Fresnel zones, changing signs as they interfere constructively or destructively at the receiver. Intuitively, the rapidly oscillating outer shells change sign over distances that are small compared with the length scales of change in the velocity profile. Their effect on the result will tend to zero and it is resonable to expect that a more restricted region around the centerline will be the effective sample volume. That intuition is born out by both calculation and experiment [2, 3].

Solution of Equation 1, either analytically or numerically, requires extensive prior manipulation to produce a form that is mathematically tractable. Defining the bounds of integration, in particular, requires careful consideration of the physical processes, including acoustic interaction with the bottom, and the mathematics [3]. The result is Equation 2, defined now in the numerical integration space (NIS) to which the eikonal has been mapped. The overbar indicates that the steady and wave components of the flow have been retained while excluding the turbulent component.

$$\overline{\psi'}(z) = \frac{kL\sqrt{2}(1+i)}{\sqrt{\pi}} \int_{x_1=0}^{1} \int_{z_1=\sqrt{\frac{k}{2L}}\frac{g(L(1-x_1^2),z)-z}{x_1}}^{\sqrt{\frac{k}{2L}}\frac{f(L(1-x_1^2),z)-z}{x_1}} \cdot \overline{n'}\left(\sqrt{\frac{2L}{k}}x_1 z_1 + z\right) x_1 e^{iz_1^2} dz_1 dx_1 \quad (2)$$

The original physical domain of integration has been stretched and flipped by these manipulations. x_1 is still the along-axis variable, however, it runs from receiver to transmitter now and has been mapped from a linear to a square-root distribution. z_1 is still the vertical cross-axis variable. Those manipulations were symmetrical about the centerline. The exponential still oscillates strongly as distance from the centerline increases. The separable integration over the horizontal cross-axis variable, y_1, was evaluated independently, producing a complex, multiplicative constant. The eikonal, like the velocity field, is now purely a function of height above the bed. The bounds of that integration depend on the finite duration of the signal, energy distribution in the acoustic field, and proximity to the bottom. That information is contained in the functions f and g. An analytic solution is possible for both uniform and more general linear velocity profiles in the absence of a boundary. In these cases the integral collapses cleanly to the velocity along the centerline [3]. This is completely consistent with both physical and mathematical intuition.

Unfortunately, boundary layer flow, the intended target of the BASS Rake, does not exhibit linear velocity profiles and intuition strongly suggests that measurement bias will exist for non-linear profiles. Consider, for example, the logarithmic profile observed in steady flow over a fixed bed.

$$u(z) = \frac{u_*}{\kappa}\ln\left(\frac{z}{z_o}\right), \quad (3)$$

This case requires a numerical solution. The variables describing the flow were assigned the values $u_* =$

$1\,cm \cdot s^{-1}$ and $z_o = 0.002\,cm$. Von Karman's constant, κ, has a value of 0.4. This corresponds to a steady flow over a flat bed of $250\,\mu m$ sand grains with a reference velocity of roughly $20\,cm \cdot s^{-1}$ at a height of $1\,m$. First, for illustrative purposes, consider wide beam transducers operating at $0.85\,MHz$ with a diameter of $2.5\,mm$. The real and imaginary parts of the integrand of Equation 2 for these conditions are shown in Figure 1.

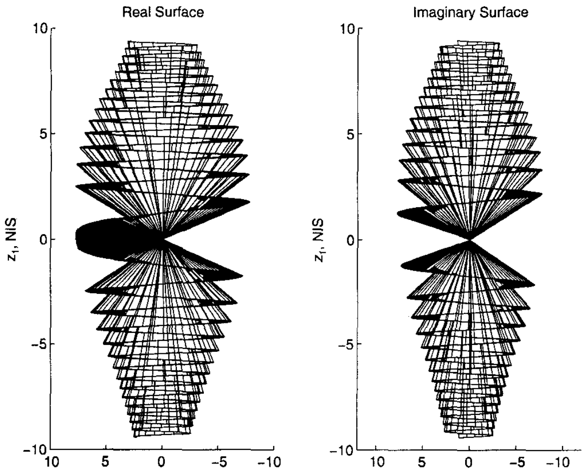

Figure 1. REAL AND IMAGINARY PARTS OF THE EIKONAL INTEGRAND - This is an NIS end view along the centerline with the transducers 5 *cmab*. Note the mild asymmetry caused by the changing curvature of the logarithmic velocity profile. The rapid oscillations of the exponential, the mathematical manifestation of the concentric Fresnel shells, and the relative importance of the central portion of the Fresnel pattern, the effective sample volume, are readily apparent.

Now consider the narrow beam, $1.75\,MHz$, $6\,mm$ transducers that are more typically used in a BASS sensor. Using these transducer characteristics and the flow parameters above, Equation 2 was solved numerically for 30 values of z, logarithmically spaced from $1\,mmab$ to $10\,cmab$. That solution is shown in Figure 2.

The flow variables control the shape of the profile, but do not affect the physical extent of the Fresnel zones. Those depend on the diameter and operating frequency of the transducers. Therefore, deviations of the calculated result for this arbitrary profile graphically show both the radius of the effective sample volume and the effect of proximity to the bottom for a given transducer [3].

First, consider that fluctuations in propagation speed caused by the velocity field should be small and not produce large relative changes in the amplitude of the acoustic signal; the real part of the eikonal should be near zero. Figure 2 confirms this intuition for the flow well above the bottom. Note, however, the increase in the amplitude below $1\,cmab$. This occurs as the central region of the Fresnel pattern begins to intersect the bottom. The effect is observed experimentally. More importantly, consider the behavior of the imaginary part of the eikonal, which is expected to track the velocity. For the $1.75\,MHz$, $6\,mm$ transducers used in this calculation, significant measurement bias, $\approx 20\,\%$, occurs only below $4\,mmab$. Above that height the measurement can be treated as unbiased.

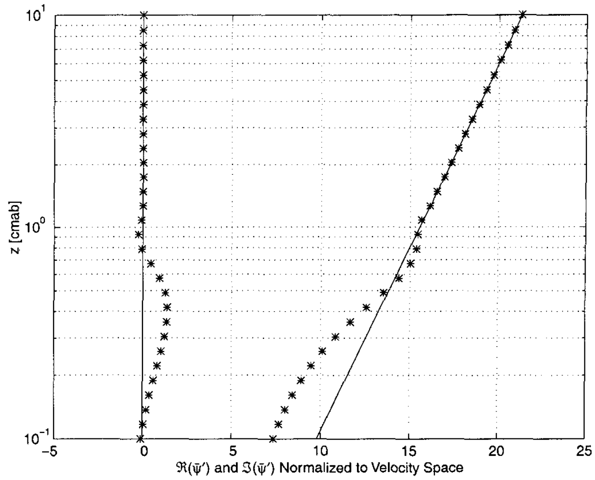

Figure 2. REAL AND IMAGINARY PARTS OF THE EIKONAL - The $*$s mark the real and imaginary parts of the eikonal normalized to velocity space. The solid lines mark the logarithmic velocity profile and zero. Significant measurement bias exists only within millimeters of the bottom. The bias is systematic and could be removed in post-processing.

Note that the closest these transducers can be placed to the bottom and still function is $3\,mm$; only the lowermost measurement in a BASS Rake profile using these transducers would exhibit bias. Other frequency/diameter combinations with the narrow far field main lobe used by BASS instruments can be placed closer to the bottom and, as here, only the lowermost level experiences significant bias [3].

A more detailed examination of the curves shows the small changes that occur as the first fringe outside the central lobe of the Fresnel pattern intersects the bottom. As the centerline continues to descend the bias is negative, as predicted, because of the changing curvature of the logarithmic profile. The absense of fluctuations higher in the curve confirms the earlier intuition that the outer fringes of the pattern would not contribute to the result and that an effective sample volume can be defined in the central region of the Fresnel pattern [3].

In closing it is worth noting that laboratory tests have demonstrated that narrow beam transducers function well right up to a natural sand bottom and that this model accurately describes their response [3]. Further, the accuracy of wave-current boundary layer models may be no greater than $20\,\%$ [1]. A measurement of this accuracy, within $1\,cm$ of the bottom, would actually be quite useful without extensive post-processing. The value of an accurate profile from $1\,cmab$ to $10\,cmab$ to the study of oceanic processes on the continental shelf would be enormous. That measurement is well within the performance envelope of $1.75\,MHz$, $6\,mm$ transducers [3].

III. SIMPLIFIED PARAMETRIC MODEL

The strength of the formal description derives from its foundation in careful modeling of the physical process. Its great weakness is computational complexity. The duration of the calculation for even a relatively simple logarithmic profile can exceed 24 hours for some transducer characteristics, effectively prohibiting extensive investigation of the effects of changing flow and transducer parameters. The Bessel functions that describe flow in a turbulent wave boundary layer would only increase the necessary computer time. A computationally simpler approximation that accurately duplicates the significant physical behavior of the effective sample volume must be developed before a more far ranging investigation of the sensor response can be performed.

Surprisingly, simply integrating the velocity profile over a fixed vertical range and averaging over that range reproduces all of the salient features of the imaginary portion of the formally derived eikonal. For any portion of the range extending below the zero crossing of the profile the velocity is defined to be zero. The vertical range depends only on the operating frequency and diameter of the transducer. It can be determined as a single fitting parameter matching the simplified model to an eikonal profile. The latter need only be determined once for each set of transducer characteristics. Empirically, however, the result for narrow beam BASS transducers is approximately 2/3 of the midpoint radius of the $\lambda/2$ Fresnel ellipse for the transducers (λ is the acoustic wavelength). That radius is easily derived by hand from the geometry. While not based on the physics of acoustic propagation, this approach is consistent with the formally derived and empirically observed result that only the central portion of the Fresnel pattern contributes significantly to the received signal. Importantly, predictive calculation of measured velocity profiles from canonical boundary layer velocity profiles using the simple parametric model is up to four orders of magnitude faster than the eikonal based computation [3].

A comparison of eikonal profiles with a suite of parametric profiles is shown in Figure 3. The three eikonal profiles required more than 35 aggregate hours of processor time. For comparison, the five parametric profiles required more processor time to plot than to calculate. Patently, the parametric profiles reproduce the behavior of the eikonal calculations to a high degree of accuracy. Differences between the two calculations are smaller than the measurement accuracy of the BASS Rake and well below the accuracy of boundary layer flow models. Profile matching indicates effective sample volume radii of $6\,mm$ for $0.85\,MHz$, $2.5\,mm$ transducers, $4\,mm$ for $1.75\,MHz$, $6\,mm$ transducers, and $4\,mm$ for $5\,MHz$, $2.5\,mm$ transducers [3].

The speed and accuracy of the parametric model make it a valuable design tool. The measurement response of candidate transducers applied to target boundary layer velocity profiles can be calculated in advance. This makes it possible to specify mechanical and electrical characteristics of a BASS Rake sensor with confidence and without extensive prototyping.

IV. SELECTING TRANSDUCERS FOR THE CONTINENTAL SHELF WBBL

The flow in laminar and turbulent wave bottom boundary layers is described by a number of published models [1, 3]. For brevity, two transducer types, $1.75\,MHz$, $6\,mm$ and $5\,MHz$, $2.5\,mm$, and a single example of a

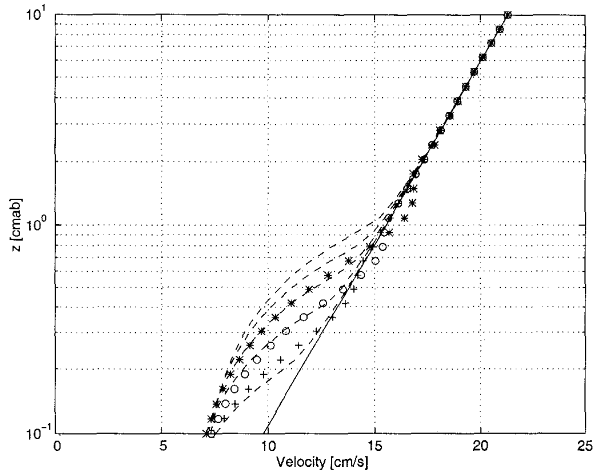

Figure 3. EIKONAL AND PARAMETRIC PROFILES - The solid line marks the logarithmic velocity profile. The dashed lines mark measurement profiles calculated with the parametric model for $r = 2\,mm$ to $r = 1\,cm$ in $2\,mm$ increments. The divergence of the parametric profiles from the actual velocity increases with r. The $*$s, $\circ$s, and $+$s mark eikonal based profiles for three sets of transducer characteristics. Setting the parametric radii to $r_* = 6\,mm$, $r_\circ = 4\,mm$, and $r_+ = 2\,mm$ reproduces the eikonal calculations to a high degree of accuracy.

turbulent wave boundary layer are considered here. The vertical resolution of the BASS Rake depends on both the vertical spacing of the transducers and on the averaging effect of the sample volume on that profile. Flow disturbance grows with the diameter of the tines on which the transducers are mounted. The disturbance is within acceptable limits for tine diameters at or below $1\,cm$. The vertical spacing of $6\,mm$ transducers in such a tine can be no less than $6\,mm$ between centers. Measurements can be made down to $3\,mmab$. The $2.5\,mm$ transudcers can be arranged in three offset columns in a $1\,cm$ tine to achieve $1\,mm$ vertical spacing with measurements starting $1.25\,mmab$ [3].

The turbulent WBBL considered here was calculated using the model of Grant and Madsen [1]. A water depth of $10\,m$ above a flat bed of $250\,\mu m$ sand grains is assumed. The model's boundary layer is produced by remotely generated swell propagating across the shelf with a period of $8\,s$ and a local amplitude of $1\,m$. These are not atypical shelf conditions. Near bed velocity profiles for these conditions are plotted in Figures 4 and 5. The profiles drawn show the velocity field at nine evenly spaced values of the phase values spanning one half of a wave period. Note particularly the restricted vertical extent of this boundary layer. The range $0.5\,cm \le \delta_w \le 2.5\,cm$ is typical for the shelf over a broad range of conditions [1, 3].

In addition to the model velocity profiles, each figure shows the measurement bias, calculated using the parametric model, as a function of height above bottom and the projected locations of transducers in a physical instrument. The larger transducers will be able to make accurate measurements down to $\approx 5\,mmab$. Data from the lowermost measurement level would require some care during post-processing and analysis, but is still quite ac-

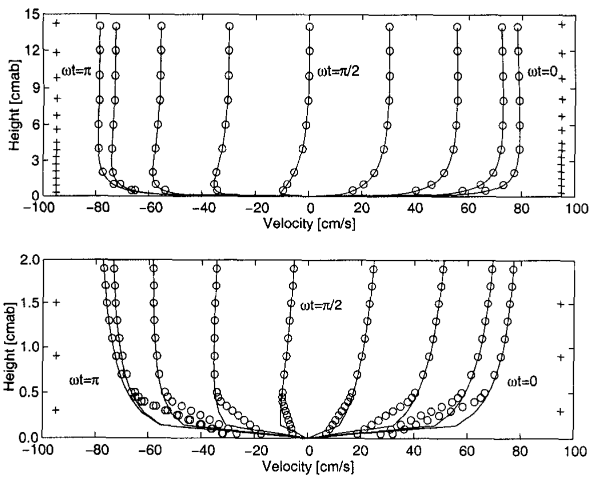

Figure 4. RESOLVING A TURBULENT WAVE BOUNDARY LAYER WITH $1.75\,MHz$, $6\,mm$ TRANSDUCERS - The solid lines are the turbulent velocity profiles at nine values of the phase, ωt. The evenly spaced values span $\omega t = 0$ to $\omega t = \pi$. The os show the measured velocity profiles for each value of ωt predicted by the parametric model. The +s along the edge of the plot mark the actual measurement heights. The upper frame shows the turbulent wave boundary layer structure up to $15\,cmab$ and the lower frame provides a detailed view of the region within $2\,cm$ of the bottom. The vertical structure of the turbulent wave boundary layer might be resolved using these transducers, but detailed measurements would not be possible.

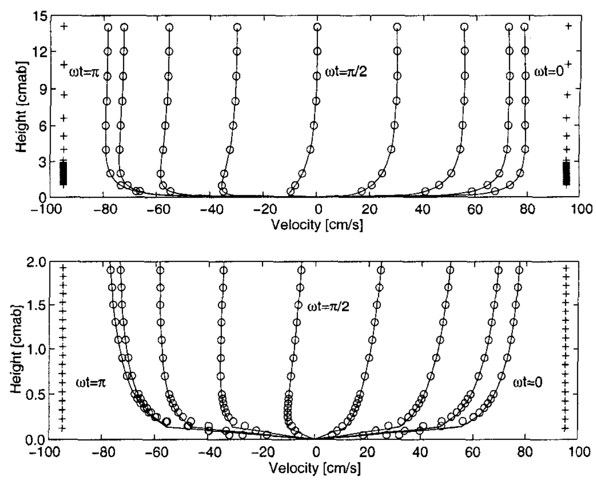

Figure 5. IMAGING A TURBULENT WAVE BOUNDARY LAYER WITH $5\,MHz$, $2.5\,mm$ TRANSDUCERS - Detailed, accurate measurements of the vertical structure of a turbulent continental shelf wave bottom boundary layer could be made with these transducers. The notation is defined in the caption of the previous figure.

curate. The $6\,mm$ vertical spacing would allow coarse resolution of the thicker turbulent wave boundary layers, such as this one. It would be difficult to do more than detect thinner boundary layers, a category that includes most laboratory and laminar cases, using these transducers. Detailed imaging of wave boundary layer flow would not be possible. However, $6\,mm$ transducers would be quite suitable for velocity profile measurements over irregular bottoms with their longer vertical length scales.

The $5\,MHz$ transducers provide both more detail and greater accuracy in resolving the vertical structure of the wave boundary layer. That there would be some improvement was easily predictable at a qualitative level. However, the implication of this calculation is not simply that a shift to $5\,MHz$ would provide significantly greater resolution. It can also be shown that a shift to $10\,MHz$ or $20\,MHz$, frequencies that would present enormous technical challenges to implement, is not necessary [3]. The continental shelf wave bottom boundary layer could be imaged in detail with transducers that are already inside the BASS Rake technology envelope. That result demonstrates the importance of the parametric model as a design tool.

V. SUMMARY

The investigation into the BASS Rake sample volume was unexpectedly fruitful. The study was initially motivated by the fear that Fresnel averaging would be too severe to permit accurate near bottom measurements. The formal eikonal model validated the empirical laboratory and field results, showing that the induced errors are, in general, negligible and, in all cases, systematic. The important serendipitous benefit has been the development of an accurate parametric model of velocity averaging over the effective sample volume. This model has become a valuable and effective tool for data evaluation and system design.

VI. ACKNOWLEDGEMENTS

I would like to express my thanks to Gene Terray of WHOI. Gene introduced me to the mathematics of the eikonal and, through numerous conversations and blackboard sessions, guided me towards the solution presented here.

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